On the Deflection of the Orbits of Earth Colliding Asteroids

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Abstract

An investigation is made of the orbital deflection of an asteroid for a collision with the Earth to be avoided. A set of equations are derived which describe the separation between the original and deflected orbits and the equations are integrated from perihelion where a velocity change is given in the direction of the orbital motion. For a predicted collision to be avoided, the separation ought to be greater than the Earth radius at the time when the asteroid crosses the Earth' orbit. It is found that if the lead time from perihelion is denoted by t measured in years, the required velocity change is $0.22(t/10)^{-1}$ cm/sec for a (semi-major axis) =2 AU and $0.17(t/10)^{-1}$ cm/sec for a=3 AU for eccentricities greater than 0.7. These values are somewhat less than the estimates obtained in earlier investigations.

Key words: asteroids-collision with earth-orbit deflection.

1. Introduction

Thanks to the works of geologists, astronomers and physicists over the past two decades, it has been established that the Earth has been subjected to the collisions of comets and asteroids. The impacting projectiles range from large comets to small asteroids (meteorites). These impacts have probably played significant roles in the evolution of fauna by killing a large number of species.

One of the implications of the research is the recognition that if such an impact should occur in the near future, not only the species including the humans living in the vicinity of the epicenter are destroyed, but the human society may be brought into a primitive society (Chapman & Morrison 1994), if the impactor were large enough (1 km in diameter or greater). Among impactors, the asteroids most frequently hit the earth, while long-periods comets hit the earth once in tens of millions of years, although the effect may be worldwide and a large number of species including humans may become extinct. Hence it is the asteroids which demand most attention.

A number of methods have been proposed to avoid such a catastrophe should an impacting body has been detected. It seems that the deflection of the orbit of the impactor is the most realistic. The object of the present paper is to estimate the change in velocity needed to deflect the orbit of the impacting body. It may be mentioned that nuclear explosion near the surface of an asteroid (Simonenko et al. 1994, Shafer et al. 1997, Yabushita 1998a) is the most effective, provided that the use of such a device in space is politically acceptable.

To estimate the required velocity, what is needed is the separation of the two bodies, one the asteroid and the other being a hypothetical body such that at a certain time, it has the same position as the asteroid but a slightly different velocity. Since the orbits are ellipses, the separation could be calculated in terms of the orbital elements. The calculation, however is somewhat complicated. Instead, we will derive a set of differential equations which describe the separation and these equations are integrated numerically to obtain the velocity required for avoiding the collision. It is appropriate to mention at this stage that once the required velocity is assessed, it is possible to estimate the yield (energy generated) of the nuclear device to be detonated at the asteroidal surface.

2. Basic equations

Let (x, y, z) be the rectangular coordinates with origin at the sun. The coordinates of a colliding asteroid is denoted by (x_0, y_0, 0) and those of the deflected orbit by (x, y, z).

If X =x - x_0 and so on, we have the differential equation

$$\ddot{X} = -GM_0\left\{\frac{X + x_0}{\left|X + x_0\right|^3} - \frac{x_0}{\left|x_0\right|^3}\right\} \quad (2.1)$$

where G is the gravitational constant and M_0 is the solar mass. Since |X|«|x|, |X|«|x_0|, where X =(X, Y, Z) and so on, we may regard |X| as a small quantity of the first order and we retain only the first order quantities in the equation (2.1). We then have

$$\left.\begin{aligned}
\ddot{X} &= GM_0\frac{(2x_0^{\ 2} - y_0^{\ 2} - z_0^{\ 2})X + 3x_0y_0Y + 3x_0z_0Z}{R^5}\\
\ddot{Y} &= GM_0\frac{3x_0y_0X + (-x_0^{\ 2} + 2y_0^{\ 2} - Z_0^{\ 2})Y + 3y_0z_0Z}{R^5}\\
\ddot{Z} &= GM_0\frac{3x_0z_0X + 3y_0z_0Y + (-x_0^{\ 2} - y_0^{\ 2} + 2z_0^{\ 2})Z}{R^5}
\end{aligned}\right\} \quad (2.2)$$

where R is the heliocentric distance, namely, $R = (x_0^{\ 2} + y_0^{\ 2} + z_0^{\ 2})^{\frac{1}{2}}$.

We also have the usual relations for Keplerian orbits;

$$\begin{aligned}
nt = E - e\sin E,\ n^2a^3 &= GM_0\\
x_0 &= a(\cos E - e)\\
y_0 &= a(1 - e^2)^{\frac{1}{2}}\sin E\\
z_0 &= 0
\end{aligned}$$

where a, e and E denote the semi-major axis, eccentricity, and the eccentric anomaly, respectively.

If the above differential equation is integrated, |X| can be plotted against time. In practice, it is easier to adopt the eccentric anomaly as the independent variable.

$$\left.\begin{aligned}
\frac{dt}{dE} &= \frac{1 - e\cos E}{n}\\
\frac{d^2x}{dE^2} &= a^3(1 - e\cos E)^2\,\frac{(2x_0^{\ 2} - y_0^{\ 2})X + 3x_0y_0Y}{R^5} + \frac{e\sin E}{1 - e\cos E}\cdot\frac{dX}{dE}\\
\frac{d^2Y}{dE^2} &= a^3(1 - e\cos E)^2\,\frac{3x_0y_0X + (-x_0^{\ 2} + 2y_0^{\ 2})}{R^5} + \frac{e\sin E}{1 - e\cos E}\cdot\frac{dY}{dE}\\
\frac{d^2Z}{dE^2} &= -a^3(1 - e\cos E)^2\,\frac{Z}{R^3} + \frac{e\sin E}{1 - e\cos E}\cdot\frac{dZ}{dE}
\end{aligned}\right\} \quad (2.3)$$

These equations are integrated numerically to calculate (X, Y, Z) and hence the separation between the orbit of the impacting asteroid and the deflected orbit.

3. Result

We consider the case where an impulse to the orbit is given at perihelion in the direction of the orbital motion, because the orbit deflection is the most effective when a sudden change in velocity is given there in the direction of the orbital motion. The separation |X| between the original and the deflected orbits is plotted against time in Figs. 1-4 for a number of asteroidal orbits. In the figures, δr stands for the separation expressed in units of the earth radius and a and e stand for the semi-major axis and eccentricity of the original orbit, respectively. Again, a cross denotes the separation at the times when the asteroid crosses the earth orbit, namely the predicted times of collision.

In order for an asteroid to avoid the collision, the separation at the time of the crossing ought to be greater than one earth radius. The figures may thus be used to estimate the required velocity change as follows.

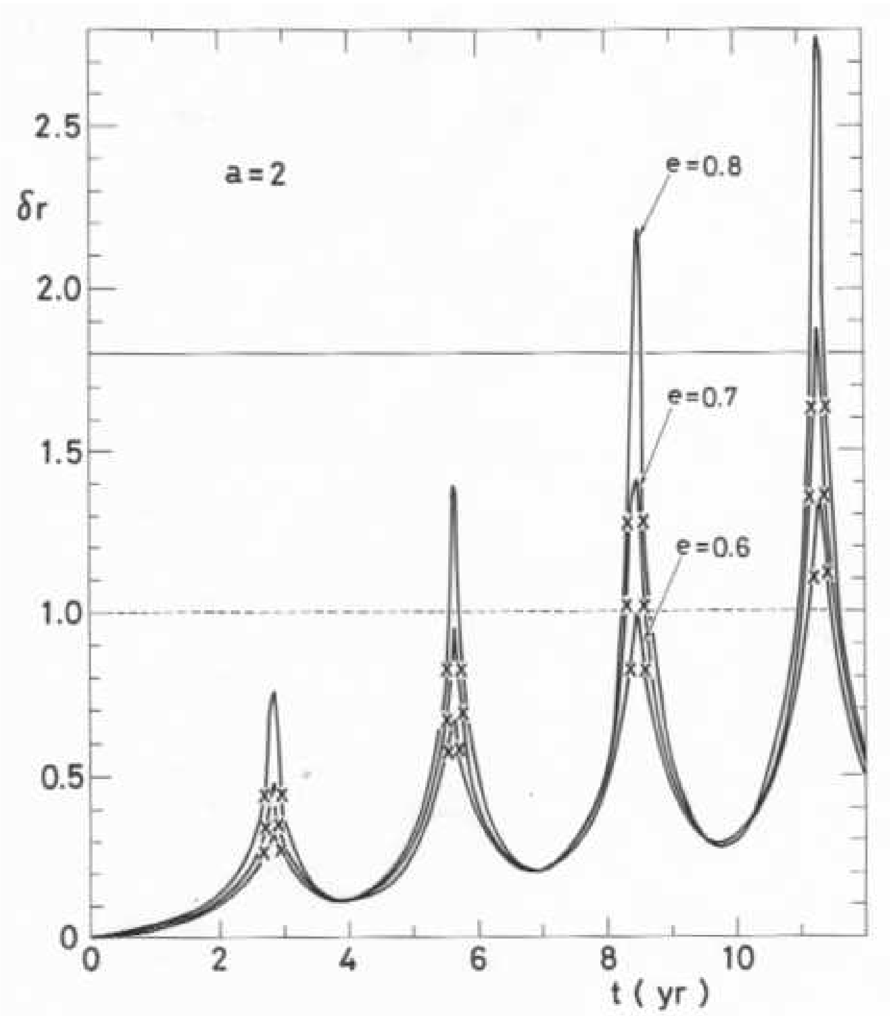


Fig. 1

The separation δr between the original and the perturbed orbits is plotted against time. δr is given in units of the earth radius and the time is in years. The velocity change given at perihelion is 0.20 cm/sec.

Take Fig. 1 as an example, where a=2 AU and e =0.8, 0.7 and 0.6. The velocity change given at perihelion is 0.20 cm/sec. Let the predicted time of collision be 8.5 years from the perihelion passage. The separation δr at the predicted time of collision is 0.8, 1.0, and 1.3 earth radii, respectively for e =0.6, 0.7 and 0.8 sec. It follows that for the case of e =0.6, the collision cannot be avoided.

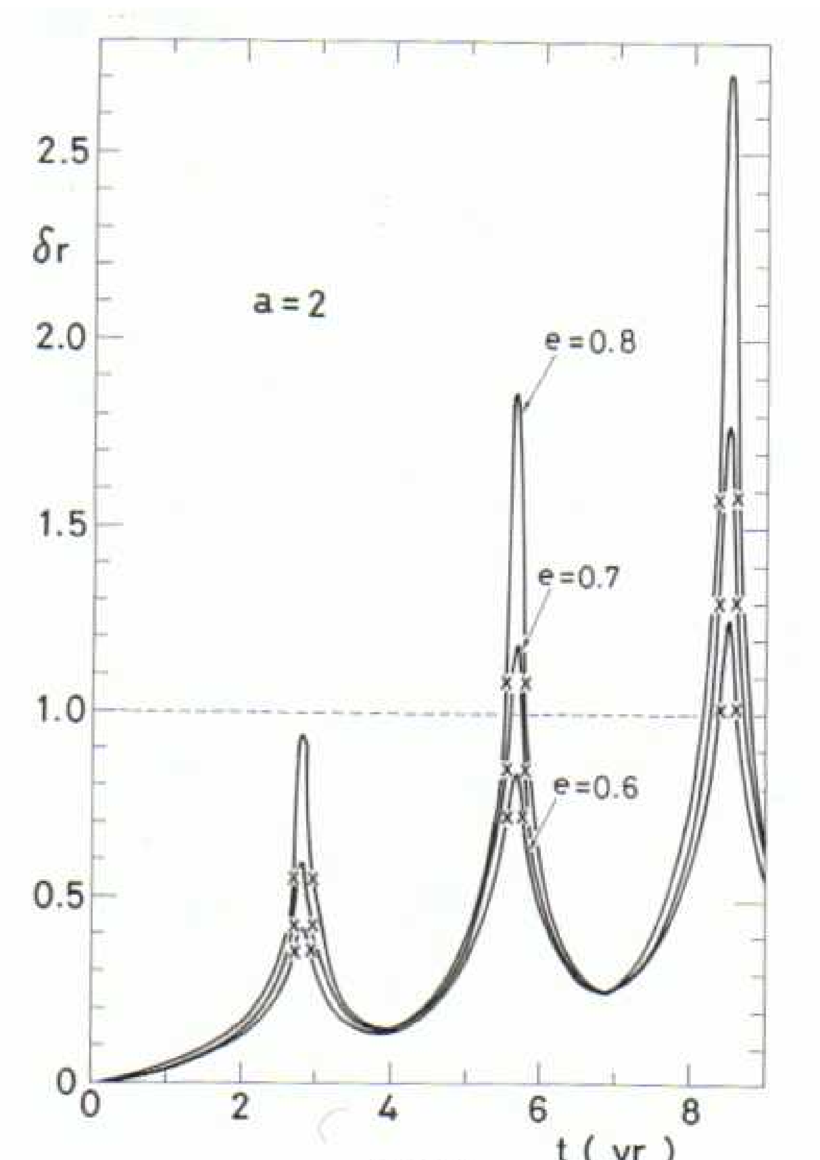


Fig. 2

The same as Fig.1 except that the given velocity change is 0.25 cm/sec.

On the other hand, Fig.2 gives the similar result for the case where the given velocity change is 0.25 cm/sec. In this case, the collision can marginally be avoided. For the cases of e =0.7 and 0.8 and in view of giving sufficient safety margin, the velocity change in excess of 0.3 cm/sec ought to be given to the asteroid for the cases of e =0.7 and 0.8.

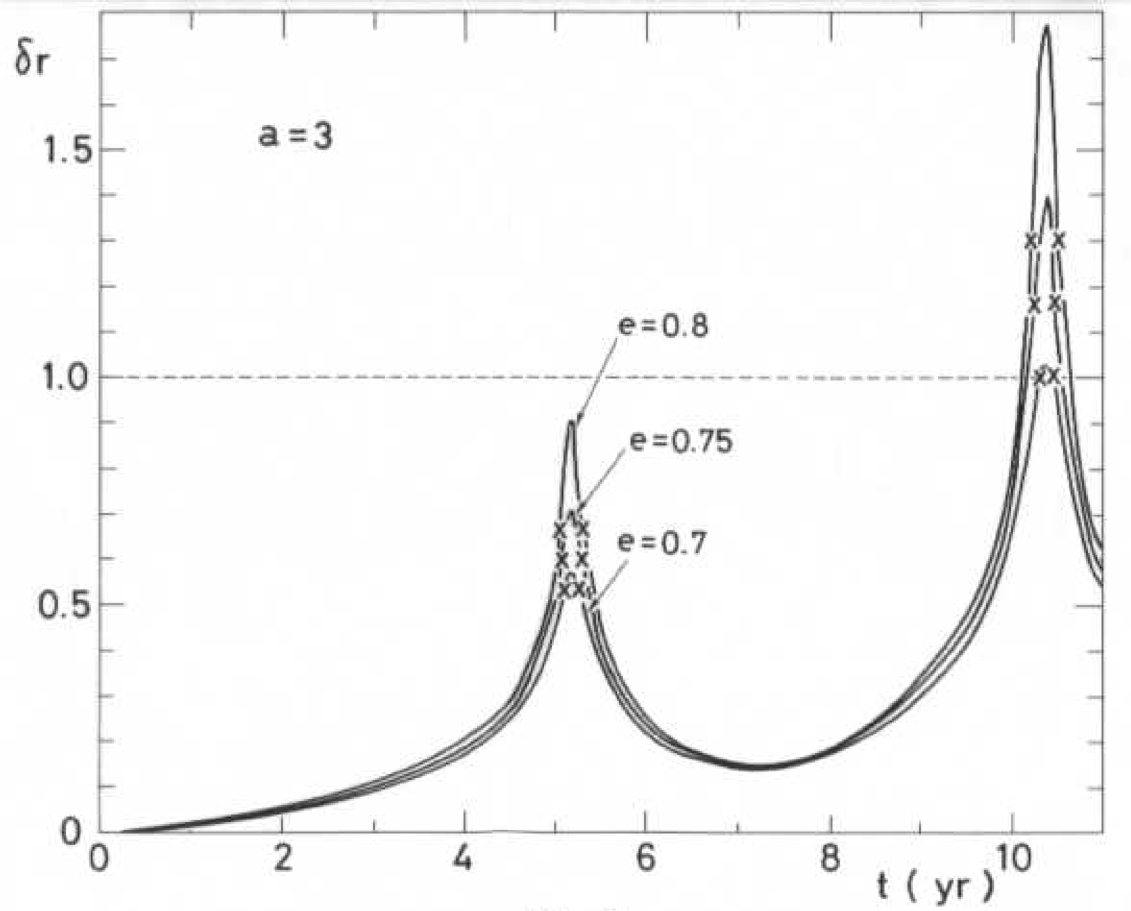
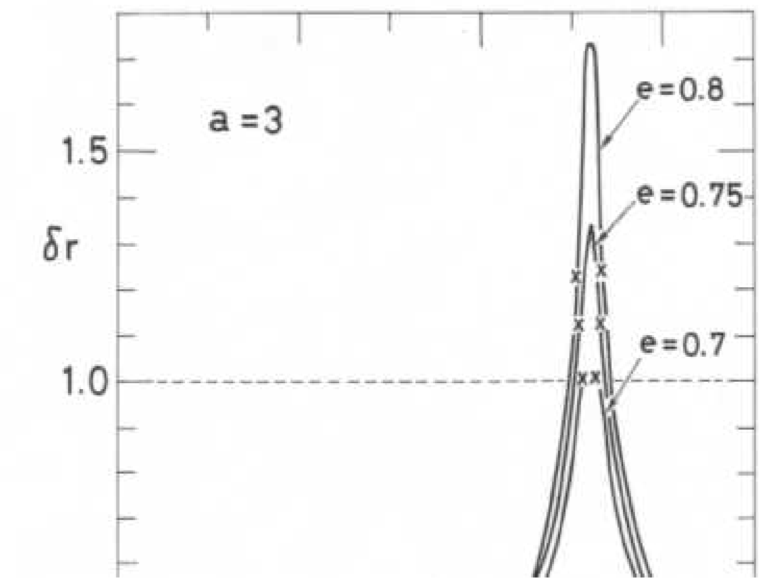


Fig. 3

The separation between the original and the perturbed orbits is plotted. The original orbit has a=3 AU. The separation is given in units of the earth radius. The velocity change given at perihelion is 0.13 cm/sec.



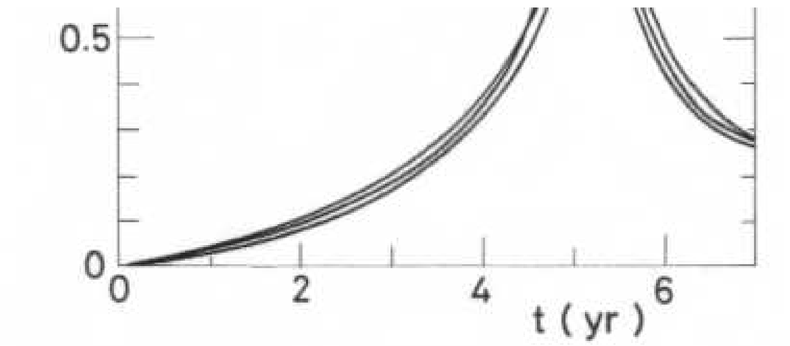


Fig. 4

The same as Fig. 3 except that the given velocity change is 0.25 cm/sec.

Similarly, the case of a =3 AU can be investigated by means of Figs.3 and 4. If the predicted time of collision is 5.2 yr from perihelion, the required velocity change at perihelion is greater than 0.25 cm/sec for the case of e =0.7 and the collision can marginally be avoided for e greater than 0.7 if the velocity change given at perihelion is 0.25 cm/sec.

4. Discussions

We are now able to obtain quantitative estimate of the required velocity change when the lead time t of the collision is given. In most cases, the lead time for collision is 10 years or longer. Ahrens&Harris (1994) derived the relation $\delta v \sim 0.7\left(\frac{t}{10}\right)^{-1}$ cm/sec where v is the required change in velocity and t is expressed in years. On the other hand, Yabushita (1998b) derived a slightly more accurate estimate, according to which the factor 0.7 in the above expression ought to be replaced by 0.75 ± 0.22 for the eccentricity equal to 0.6 or smaller. For smaller eccentricity, a greater value of v is required, which is in accord with our result presented here. If the expression derived therein is extrapolated, one finds that the required velocity change is 0.45 cm/sec for a lead time of 10 years. It may be noted that the expressions derived by Ahrens& Harris (1994) and Yabushita (1998b) are not dependent on the semi-major axis of the colliding asteroid.

We now proceed to obtaining a useful simple estimate of the required velocity change. The result presented in section 3 may be summarized as follows;

$\delta v = 0.22\left(\frac{t}{10}\right)^{-1}$ cm/sec for a =2 AU

$\delta v = 0.17\left(\frac{t}{10}\right)^{-1}$ cm/sec for a =3 AU

These estimates are valid for eccentricities greater than 0.7. We find that the estimate derived here is somewhat less than the ones obtained earlier. However, the method presented here can easily be extended to various combinations of (a, e). It is intended to carry out more extensive investigations in the future work.

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